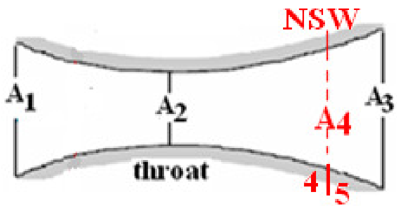
Cairo University
Faculty of Engineering
Mech. Power Eng. Dept.

Report # 2
Gas Dynamics

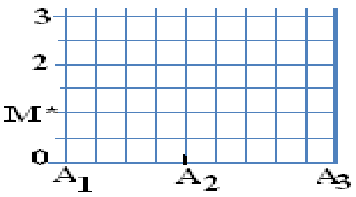
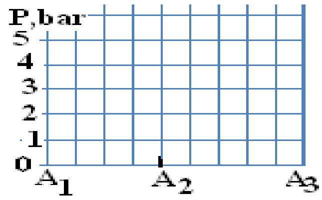
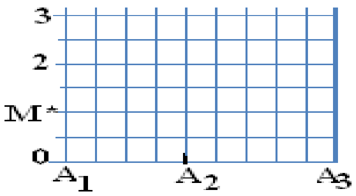
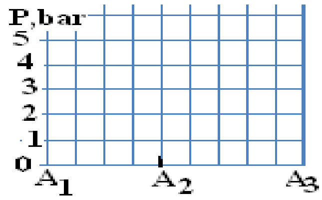
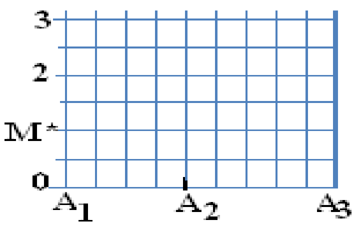
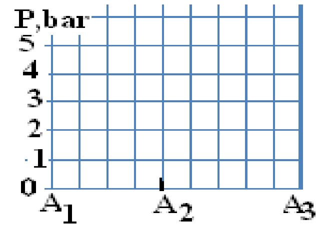
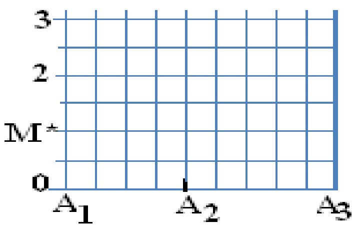
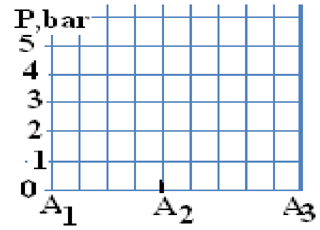
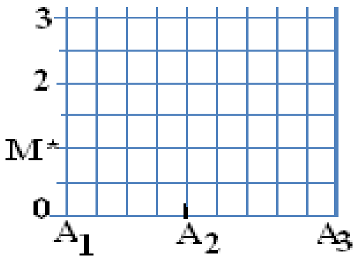
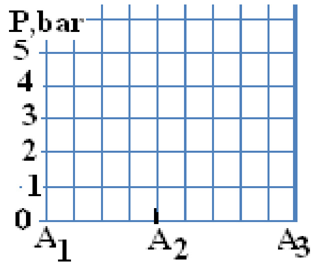
3ed Year Mechanics (Power)
2^{nd}Term 2023/2024
Fluid Mechanics Applications-2 MEP 3090

Name: ……………………………. Section: ………..

مطلوب كتابة النتائج وجميع الرسومات فى الجدول وتسليمه مع تقرير به تفاصيل الحسابات



It is required to design all the isentropic air flow conditions in a converging-diverging duct which has the following real dimensions: A_1=0.5 m^2, A_2=0.3 m^2 and A_3=0.7 m^2. The available stagnation conditions are: P_o= 7 bar & T_o=600 K. Using attached empirical equations without using any tables, graphs or trial & error, **Complete the Mach number & P design data table**:

Conditions	At inlet A_1	At throat A_2	At exit A_3	Sketch distributions of M # & P along duct axis
Case (1): Is it chocked or non-chocked flow?……… ……………	Subsonic inlet **M_1 = 0.25** A_1*=… … m^2 P_1 = …..bar *m*=…….kg/s	A_2*=… …m^2 M_2 = …….. P_2 = …..bar P*=………bar *m*=…….kg/s	Type of flow at exit………... A_3*=… …m^2 M_3= ……… P_3 = ……..bar	 
Case (2): Is it chocked or non-chocked flow?……… ……………	Supersonic inlet A_1***=as case(1)** M_1 =…….. P_1 = …..bar *m*=…….kg/s	A_2*=… …m^2 M_2 = …….. P_2 = …..bar P*=………bar *m*=…….kg/s	Type of flow at exit………... A_3*=… …m^2 M_3= ……… P_3 = ……..bar	 
Case (3): Is it chocked or non-chocked flow?……… ……………	Subsonic inlet **M_1 = 0.381** A_1*=… … m^2 P_1 = …..bar *m*=…….kg/s	A_2*=… …m^2 M_2 = …….. P_2 = …..bar P*=………bar *m*=…….kg/s	**Subsonic exit at** M_3 = …… P_3 = ……..bar **Supersonic exit** M_3 = …… P_3 = ……..bar	 
Case (4): Is it chocked or non-chocked flow?……… ……………	**Supersonic inlet** **M_1 = 1.9798** A_1*=… … m^2 P_1 = …..bar *m*=…….kg/s	A_2*=… …m^2 M_2 = …….. P_2 = …..bar P*=………bar *m*=…….kg/s	**Subsonic exit at** M_3 = …… P_3 = ……..bar **Supersonic exit** M_3 = …… P_3 = ……..bar	 
Case(5)same inlet as in **case(3)**above but we have **normal shock wave** at **A_4=0.5**m^2, Is flow chocked or not?............. M_2=…., A_1*=A_2*=A_4*=….. m^2& *m*=…….kg/s **Before NSW**: M_4=…….., P_4= …….bar, Po_4=…..….bar **After N.S.W:**M_5=……. , P_5=………bar, Po_5=…….bar , Po_3=……bar,A_5*/A_4*=………&M_3 = ……P_3 = ……..bar				 

T-S diagram of case-1	**T-S diagram of case-2**	**T-S diagram of case-3**	**T-S diagram of case-4**	**T-S diagram of case-5**

Some Useful Gas Dynamics Relations

Stagnation Reference States:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \;..........\;..., \quad \frac{P_o}{P} = \{1 + \frac{\gamma - 1}{2} M^2\}^{\frac{\gamma}{\gamma - 1}}$$

For Isentropic Flow in a Variable Area Duct :

$$\frac{A}{A*} = \frac{1}{M}\left(\frac{2 + (\gamma - 1)M^2}{(\gamma + 1)}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} ...., \frac{A}{A*} = \frac{(1 + 0.2M^2)^3}{1.728 * M} .... for \longrightarrow \gamma = 1.4$$

$$M \approx \frac{1 + 0.27\,(A/A*)^{-2}}{1.728\,(A/A*)} .... for .subsonic \;.flow ... \gamma = 1.4 ... and \longrightarrow 1.34 < A/A* < \infty$$

$$M \approx 1 - 0.88\{\ln(A/A*)\}^{0.45} .... for .subsonic \;.flow ... \gamma = 1.4 ... and \longrightarrow 1.0 < A/A* < 1.34$$

$$M \approx 1 + 1.2(A/A* - 1)^{1/2} .... for .supersonic \;.flow ... \gamma = 1.4 ... and \longrightarrow 1.0 < A/A* < 2.9$$

$$M \approx \left[216\,(A/A*) - 254\,(A/A*)^{2/3}\right]^{1/5} .... for .supersonic \;.flow ... \gamma = 1.4 ... and \longrightarrow 2.9 < A/A* < \infty$$

For Normal shock Wave :

$$\frac{P_2}{P_1} = \frac{1}{\gamma + 1}\left[2\gamma M_1^2 - (\gamma - 1)\right] = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} ...., \quad M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2} ...., \quad \frac{T_2}{T_1} = \left[2 + (\gamma - 1)M_1^2\right]\frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)^2 M_1^2}$$

$$\frac{P_{o2}}{P_{o1}} = \frac{\rho_{o2}}{\rho_{o1}} = \left[\frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}\right]^{\frac{\gamma}{\gamma - 1}}\left[\frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma - 1}} ...., \quad \frac{A*_2}{A*_1} = \frac{M_2}{M_1}\left[\frac{(\gamma - 1)M_1^2 + 2}{(\gamma - 1)M_2^2 + 2}\right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

For Fanno-Line Flow :

$$\frac{\bar{f}.L*}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma}\ln\left[\frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2}\right] ...., \quad \frac{\bar{f}.\Delta L}{D} = \left(\frac{\bar{f}.L*_1}{D}\right) - \left(\frac{\bar{f}.L*_2}{D}\right)$$

$$\frac{T}{T*} = \frac{a^2}{a*^2} = \frac{\gamma + 1}{2 + (\gamma - 1)M^2} ...., \quad \frac{P}{P*} = \frac{1}{M}\left[\frac{\gamma + 1}{2 + (\gamma - 1)M^2}\right]^{1/2}$$

$$\frac{\rho}{\rho*} = \frac{V*}{V} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{1/2} ...., \quad \frac{P_o}{P_o*} = \frac{\rho_o}{\rho_o*} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

For Isothermal Flow with Friction:

$$\frac{\bar{f}.L_{max}}{D} = \frac{1 - \gamma M^2}{\gamma M^2} + \ln(\gamma M^2) ...., \quad \frac{P_1}{P'} = \frac{1}{M_1.\gamma^{0.5}} ...., \quad \frac{V_1}{V'} = \frac{\rho'}{\rho_1} = M_1.\gamma^{0.5}$$

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{M_2}{M_1} ...., G^2 = (\dot{m}/A)^2 = \frac{P_1^2 - P_2^2}{R.T\left[(\bar{f}.L/D) + 2\ln(P_1/P_2)\right]}$$

For Rayliegh Line Flow :

$$\frac{T_o}{T_o*} = \frac{(\gamma + 1)M^2\left[2 + (\gamma - 1)M^2\right]}{(1 + \gamma.M^2)^2} ...., \quad \frac{T}{T*} = \frac{(\gamma + 1)^2.M^2}{(1 + \gamma.M^2)^2} ...., \quad \frac{P}{P*} = \frac{\gamma + 1}{1 + \gamma.M^2}$$

$$\frac{V}{V*} = \frac{\rho*}{\rho} = \frac{(\gamma + 1)M^2}{1 + \gamma.M^2} ...., \quad \frac{P_o}{P_o*} = \frac{\gamma + 1}{1 + \gamma.M^2}.\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{\frac{\gamma}{\gamma - 1}}$$

Good Luck – Dr. Mohsen Soliman